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Technical Note

A general bioheat model at macroscale

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ABSTRACT

We develop a general bioheat transport model at macroscale for biological tissues with the required closure provided. The model shows that both blood and tissue macroscale temperatures satisfy the dual-phase-lagging (DPL) energy equations. Due to the coupled conduction between the blood and the tissue, thermal waves and possibly resonance may appear in bioheat transport. The blood–tissue interaction yields a very rich effect of the interfacial convective heat transfer, the blood velocity, the perfusion and the metabolic reaction on blood and tissue macroscale temperature fields. Examples include: (i) the spreading of tissue metabolic effect into the blood DPL bioheat equation, (ii) the appearance of the convection term in the tissue DPL bioheat equation due to the blood velocity, and (iii) the appearance of sophisticated heat source terms in energy equations for blood and tissue temperatures.

1. Introduction

Heat transport in biological tissues is enriched by heat conduction in the tissue and the vascular system, blood–tissue convection, blood perfusion, and also metabolic heat generation in tissues. Its accurate description is critical not only for fundamental understanding of biological processes/functions such as the human thermoregulation [1], but also for many medical and biological treatments such as hyperthermia and hypothermia [2], laser surgery [3], infrared irradiation [4], hypothermic preservation and cryopreservation [5].

A microscopic study of heat transport in biological tissues is complicated because of the complicity in microscale anatomical structure of biological media. Therefore the bioheat transport is normally examined at macroscale, a phenomenological scale that is much larger than the microscale of cells and voilds and much smaller than the system length scale [6]. Macroscale bioheat models have been developed either by the mixture theory or by the porous-media theory [6]. The former views blood and tissues as a mixture of continuum deformable media and develops the macroscale point equations via scaling down the global balance equations. In this approach, neither microscale presentation of the system nor microscale quantities is introduced. Phase properties are defined at the macroscale. The global balance equations are formed in terms of macroscale properties and with additional terms accounting for the interaction between blood and tissue. Required constitutive equations for the heat flux vector are supplied by the Fourier law [7], the Cattaneo–Vernotte (CV) relation [8] or the dual-phase-lagging (DPL) relation [8,9]. The thermal models developed in this approach include the classical Pennes model [10], the Wulff model [11], the Klinger model [12], Chen and Holmes model [13], the thermal-wave bioheat model [14] and the DPL bioheat model [3,15].

The porous-media theory considers biological tissue as a blood-saturated porous matrix including cells and interstices and develops the macroscale point equations by scaling up the microscale model. In this approach, both conservation and constitutive equations are introduced at the microscale. The resulting microscale field equations are then averaged over a representative elementary volume (REV) to obtain the macroscale field equations. In the process of averaging, the multiscale theorems are used to convert integrals of gradient, divergence, curl, and partial time derivatives of a function into some combination of gradient, divergence, curl, and partial time derivatives of integrals of the function and integrals over the boundary of the REV [16]. The closure model must be provided for the unclosed terms in macroscale field equations that represent the microscale effect in order to form a closed system. A rigorous closure is not available at present [6]. Some approximate models developed by this approach include those in [5,17,18].

Simplicity is the main advantage of the mixture-theory approach. However, it offers no connection between microscale and macroscale properties and is not capable to accurately describe the rich blood–tissue interaction [6]. The porous-media approach successfully overcomes these drawbacks, thereby offering an effective way for developing accurate macroscale thermal models for biological tissues. By applying the porous-media approach,

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Nomenclature

a_v	specific heat transfer area
A_{bt}	area of the blood–tissue interface in the representative elementary volume (REV)
c	specific heat
h	blood–tissue interfacial convective heat transfer coefficient
k	thermal conductivity
k_e	effective thermal conductivity
$\mathbf{n}_{bt}$	outward-directed surface normal vector from b-phase to t-phase
$\mathbf{v}$	macroscale (intrinsic average) velocity
$\tilde{\mathbf{v}}$	spatial deviation velocity
V^b	blood volume in the REV
V^{REV}	volume of the REV
T	macroscale (intrinsic average) temperature
α	effective diffusivity
ε	volume fraction
Φ	thermal source
ρ	density
τ_q	phase lag of the heat flux
τ_T	phase lag of the temperature gradient
ω	blood perfusion rate

Superscripts

b	blood phase
t	tissue phase

we develop a relatively rigorous macroscale bioheat model that includes the effect of blood–tissue coupled conduction and is with blood or tissue temperature as the sole unknown temperature. We also examine the general features of bioheat transport with the model developed.

2. Macroscale nonequilibrium heat transport model

We describe the two phases in blood-infiltrated biological tissues as b- and t-phases, denoting the vascular system (blood phase) and the surrounding tissue (tissue phase), respectively. Assume that blood is incompressible and Newtonian. A good discussion is available in [19] regarding the validation region of this assumption. Take both the blood–tissue interfacial convective heat transfer and the blood perfusion into account. Following the same procedure of developing the two energy equation model for transport in porous media [20,21], a volume average of microscale conservation equations over a representative elementary volume (REV; Fig. 1) yields macroscale energy equations for b- and t-phases,

$$(\rho c\varepsilon)^b\left(\frac{\partial T^b}{\partial t}+\mathbf{v}^b\cdot\nabla T^b\right)-\mathbf{u}^{bb}\cdot\nabla T^b-\mathbf{u}^{bt}\cdot\nabla T^t = \nabla\cdot(\mathbf{K}^{bb}\cdot\nabla T^b+\mathbf{K}^{bt}\cdot\nabla T^t)-G^b(T^b-T^t), \tag{1}$$

$$(\rho c\varepsilon)^t\frac{\partial T^t}{\partial t}-\mathbf{u}^{tb}\cdot\nabla T^b-\mathbf{u}^{tt}\cdot\nabla T^t=\nabla\cdot(\mathbf{K}^{tb}\cdot\nabla T^b+\mathbf{K}^{tt}\cdot\nabla T^t) + G^t(T^b-T^t)+\varepsilon^t\Phi^t. \tag{2}$$

Here superscripts b and t refer to the blood and tissue phases, respectively. $\mathbf{v}$ and T are the macroscale (intrinsic average) velocity and temperature, respectively. ρ, c and ε are the density, specific heat and volume fraction, respectively. Φ is the thermal source which may come from the heat generation by metabolic reaction or the external heat supply like the one used in hyperthermia therapy. G is a blood-tissue coupling factor which represents the combined effect of both the blood-tissue interfacial convective heat transfer and the blood perfusion. The thermal dispersion tensors $\mathbf{K}^{bb}$ and $\mathbf{K}^{bt}$ are

$$\mathbf{K}^{bb}=(\varepsilon k)^b\mathbf{I}+\frac{k^b}{V^{\mathrm{REV}}}\int_{A_{bt}}\mathbf{n}_{bt}\mathbf{b}^{bb}\,dA-(\rho c)^b\langle\tilde{\mathbf{v}}^b\mathbf{b}^{bb}\rangle, \tag{3}$$

$$\mathbf{K}^{bt}=\frac{k^b}{V^{\mathrm{REV}}}\int_{A_{bt}}\mathbf{n}_{bt}\mathbf{b}^{bt}\,dA-(\rho c)^b\langle\tilde{\mathbf{v}}^b\mathbf{b}^{bt}\rangle. \tag{4}$$

The thermal conductivity tensors $\mathbf{K}^{tb}$ and $\mathbf{K}^{tt}$ are

$$\mathbf{K}^{tb}=\frac{k^t}{V^{\mathrm{REV}}}\int_{A_{bt}}\mathbf{n}_{tb}\mathbf{b}^{tb}\,dA, \tag{5}$$

$$\mathbf{K}^{tt}=(\varepsilon k)^t\mathbf{I}+\frac{k^t}{V^{\mathrm{REV}}}\int_{A_{bt}}\mathbf{n}_{tb}\mathbf{b}^{tt}\,dA. \tag{6}$$

The four velocity-like coefficients are given by

$$\mathbf{u}^{bb}=\frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}}\mathbf{n}_{bt}\cdot k^b\nabla\mathbf{b}^{bb}\,dA-\frac{k^b}{V^{\mathrm{REV}}}\int_{A_{bt}}\mathbf{n}_{bt}s^b\,dA + (\rho c)^b\langle\tilde{\mathbf{v}}^b s^b\rangle-(\rho c)^b\frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}}\mathbf{n}_{bt}\cdot\mathbf{v}^b\mathbf{b}^{bb}\,dA, \tag{7}$$

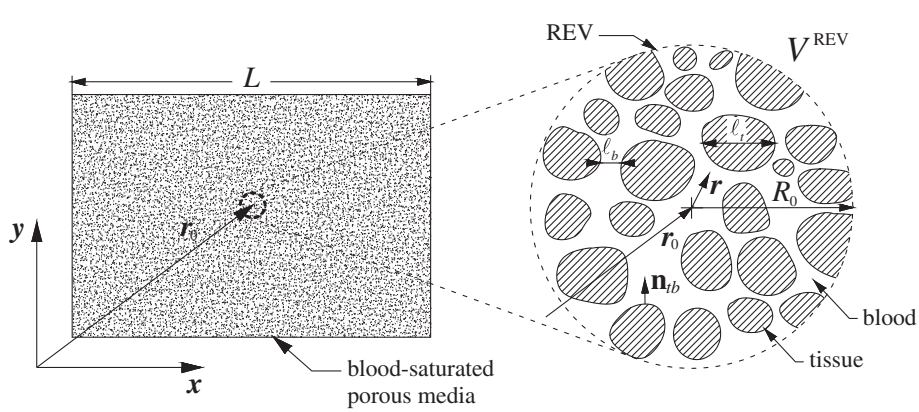

Fig. 1. Blood-saturated porous media and representative elementary volume (REV).

$$\mathbf{u}^{bt} = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot k^b\nabla\mathbf{b}^{bt}\,dA + \frac{k^b}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}s^b\,dA - (\rho c)^b\langle\tilde{\mathbf{v}}^b s^b\rangle - (\rho c)^b\frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot\mathbf{v}^b\mathbf{b}^{bt}\,dA, \tag{8}$$

$$\mathbf{u}^{tb} = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{tb}\cdot k^t\nabla\mathbf{b}^{tb}\,dA - \frac{k^t}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{tb}s^t\,dA \tag{9}$$

$$\mathbf{u}^{tt} = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{tb}\cdot k^t\nabla\mathbf{b}^{tt}\,dA + \frac{k^t}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{tb}s^t\,dA \tag{10}$$

The blood–tissue coupling factor, the lumped convection-perfusion parameter, is

$$G^b = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot k^b\nabla s^b\,dA - (\rho c)^b\frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot\mathbf{v}^b s^b\,dA. \tag{11}$$

$$G^t = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot k^t\nabla s^t dA \tag{12}$$

In Eqs. (3)–(12), k is the thermal conductivity. V^{REV} and V^b are the volume of REV and the blood volume in the REV, respectively. $\mathbf{n}_{bt}$ is the outward-directed surface normal vector from the b-phase toward the t-phase, and $\mathbf{n}_{bt} = -\mathbf{n}_{tb}$ (Fig. 1). A_{bt} is the area of the blood–tissue interface in the REV. $\tilde{\mathbf{v}}$ is the spatial deviation velocity, the difference between the microscale velocity and the macroscale (intrinsic average) velocity. $\langle\rangle$ denotes the superfacial average. $\mathbf{b}^{bb}$, $\mathbf{b}^{bt}$, $\mathbf{b}^{tb}$, $\mathbf{b}^{tt}$, s^b and s^t are the closure variables or the mapping variables that link the microscale and macroscale and are governed by the following three closure problems with $\mathbf{r}$ and $\ell_i (i = 1,2,3)$ as the position vector and the lattice vector, respectively:

Problem I:

$$(\rho c)^b\tilde{\mathbf{v}}^b + (\rho c)^b\mathbf{v}^{\mathbf{b}}\cdot\nabla\mathbf{b}^{bb} = k^b\nabla^2 b^{bb} - \frac{1}{\varepsilon^b}c^{bb},\ \text{in the } b\text{-phase}, \tag{13}$$

$$\text{B.C.1 } \mathbf{b}^{bb} = \mathbf{b}^{tb},\ \text{at } A_{bt}, \tag{14}$$

$$\text{B.C.2 } \mathbf{n}_{bt}\cdot k^b\nabla\mathbf{b}^{bb} = \mathbf{n}_{bt}\cdot k^t\nabla\mathbf{b}^{tb} - \mathbf{n}_{bt}k^b,\ \text{at } A_{bt}, \tag{15}$$

$$0 = k^t\nabla^2\mathbf{b}^{tb} + \frac{1}{\varepsilon^t}\mathbf{c}^{tb},\ \text{in the } t\text{-phase}, \tag{16}$$

$$\text{Average}: \langle\mathbf{b}^{bb}\rangle^b = 0,\quad \langle\mathbf{b}^{tb}\rangle^t = 0, \tag{17}$$

$$\text{Periodicity}: \mathbf{b}^{bb}(\mathbf{r}+\ell_i) = \mathbf{b}^{bb}(\mathbf{r}),\quad \mathbf{b}^{tb}(\mathbf{r}+\ell_i) = \mathbf{b}^{tb}(\mathbf{r}),\quad i = 1,2,3, \tag{18}$$

where

$$\mathbf{c}^{bb} = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot k^b\nabla\mathbf{b}^{bb}\,dA - (\rho c)^b\frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot\mathbf{v}^b\mathbf{b}^{bb}\,dA,$$

$$\mathbf{c}^{tb} = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot k^t\nabla\mathbf{b}^{tb}\,dA.$$

Problem II:

$$(\rho c)^b\mathbf{v}^b\cdot\nabla\mathbf{b}^{bt} = k^b\nabla^2\mathbf{b}^{bt} - \frac{1}{\varepsilon^b}\mathbf{c}^{bt},\ \text{in the } b\text{-phase}, \tag{19}$$

$$\text{B.C.1 } \mathbf{b}^{bt} = \mathbf{b}^{tt},\ \text{at } A_{bt}, \tag{20}$$

$$\text{B.C.2 } \mathbf{n}_{bt}\cdot k^b\nabla\mathbf{b}^{bt} = \mathbf{n}_{bt}\cdot k^t\nabla\mathbf{b}^{tt} + \mathbf{n}_{bt}k^t,\ \text{at } A_{bt}, \tag{21}$$

$$0 = k^t\nabla^2\mathbf{b}^{tt} + \frac{1}{\varepsilon^t}\mathbf{c}^{tt},\ \text{in the } t\text{-phase}, \tag{22}$$

$$\text{Average}: \langle\mathbf{b}^{bt}\rangle^b = 0,\quad \langle\mathbf{b}^{tt}\rangle^t = 0, \tag{23}$$

$$\text{Periodicity}: \mathbf{b}^{bt}(\mathbf{r}+\ell_i) = \mathbf{b}^{bt}(\mathbf{r}),\quad \mathbf{b}^{tt}(\mathbf{r}+\ell_i) = \mathbf{b}^{tt}(\mathbf{r}),\quad i = 1,2,3, \tag{24}$$

where

$$\mathbf{c}^{bt} = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot k^b\nabla\mathbf{b}^{bt}\,dA - (\rho c)^b\frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot\mathbf{v}^b\mathbf{b}^{bt}\,dA,$$

$$\mathbf{c}^{tt} = \frac{1}{V^{\mathrm{REV}}}\int_{A_{bt}} \mathbf{n}_{bt}\cdot k^t\nabla\mathbf{b}^{tt}\,dA.$$

Problem III:

$$(\rho c)^b\mathbf{v}^b\cdot\nabla s^b = k^b\nabla^2 s^b - \frac{1}{\varepsilon^b}G^b,\ \text{in the } b\text{-phase}, \tag{25}$$

$$\text{B.C.1 } s^b = s^t + 1,\ \text{at } A_{bt}, \tag{26}$$

$$\text{B.C.2 } \mathbf{n}_{bt}\cdot k^b\nabla s^b = \mathbf{n}_{bt}\cdot k^t\nabla s^t\mathbf{n}_{bt}\cdot\nabla s^t - \frac{(\rho c\omega)^b}{a_v},\ \text{at } A_{bt}, \tag{27}$$

$$0 = k^t\nabla^2 s^t + \frac{1}{\varepsilon^t}G^t,\ \text{in the } t\text{-phase}, \tag{28}$$

$$\text{Average}: \langle s^b\rangle^b = 0,\quad \langle s^t\rangle^t = 0, \tag{29}$$

$$\text{Periodicity}: s^b(\mathbf{r}+\ell_i) = s^b(\mathbf{r}),\quad s^t(\mathbf{r}+\ell_i) = s^t(\mathbf{r}),\quad i = 1,2,3, \tag{30}$$

where G^b and G^t are given by Eqs. (11) and (12).

ω is the blood perfusion rate. a_v is the specific heat transfer area. $\langle\rangle^b$ and $\langle\rangle^t$ denote the intrinsic average over blood and tissue volumes, respectively. The three closure problems are obtained based on the unit-cell approach [20,21]. The approach imposes the periodicity condition because the closure problem is normally solved only in some representative region that can be treated as a unit cell in a spatially periodic model. The three closure problems are thus valid for biological tissues that can be approximated by this periodic assumption. Otherwise, the lattice vector $\ell_i (i = 1,2,3)$ must cover whole biological tissues of interest so that the whole system is taken as the unit cell.

The model in Eqs. (1)–(30) is more general than those in [5,17,18]. It includes the effect of both blood–tissue coupled conduction terms [$\mathbf{K}^{bt}\cdot\nabla T^t$ in Eq. (1) and $\mathbf{K}^{tb}\cdot\nabla T^b$ in Eq. (2)] and non-traditional convective transport terms [$(-\mathbf{u}^{bb}\cdot\nabla T^b - \mathbf{u}^{bt}\cdot\nabla T^t)$ in Eq. (1) and $(-\mathbf{u}^{tb}\cdot\nabla T^b - \mathbf{u}^{tt}\cdot\nabla T^t)$ in Eq. (2)]. It also provides the rigorous closures which offer the models for microscale effects on macroscale [Eqs. (3)–(30)]. While the three closure problems appear sophisticated, they can be effectively resolved by standard numerical schemes. For the details of resolving such closure problems, the readers are referred to, for example, [22,23] that solve a similar closure problem for heat transport in nanofluids.

3. Macroscale model for blood and tissue temperatures

Rewrite Eqs. (1) and (2) in their operator form:

$$\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}\begin{bmatrix}T^b\\ T^t\end{bmatrix} = \begin{bmatrix}0\\ \varepsilon^t\Phi^t\end{bmatrix}, \tag{31}$$

where

$$\mathbf{A} = \gamma^b\left(\frac{\partial}{\partial t} + \mathbf{v}^b\cdot\nabla\right) - \mathbf{u}^{bb}\cdot\nabla - \nabla\cdot(\mathbf{K}^{bb}\cdot\nabla) + G^b, \tag{32}$$

$$\mathbf{B} = -\mathbf{u}^{bt}\cdot\nabla - \nabla\cdot(\mathbf{K}^{bt}\cdot\nabla) - G^b, \tag{33}$$

$$\mathbf{C} = -\mathbf{u}^{tb}\cdot\nabla - \nabla\cdot(\mathbf{K}^{tb}\cdot\nabla) - G^t, \tag{34}$$

$$\mathbf{D} = \gamma^t\frac{\partial}{\partial t} - \mathbf{u}^{tt}\cdot\nabla - \nabla\cdot(\mathbf{K}^{tt}\cdot\nabla) + G^t, \tag{35}$$

$$\gamma^b = (\rho c\varepsilon)^b,\quad \gamma^t = (\rho c\varepsilon)^t, \tag{36}$$

We then obtain an uncoupled form by evaluating the determinant of the operator:

$$\begin{aligned}
&\frac{\partial T^i}{\partial t}+\frac{\gamma^b\gamma^t}{G^b\gamma^t+G^t\gamma^b}\frac{\partial^2 T^i}{\partial t^2}+\frac{1}{G^b\gamma^t+G^t\gamma^b}[G^t\gamma^b\mathbf{v}^b-G^b(\mathbf{u}^{tt}+\mathbf{u}^{tb})\\
&-G^t(\mathbf{u}^{bb}+\mathbf{u}^{bt})]\cdot\nabla T^i=\frac{1}{G^b\gamma^t+G^t\gamma^b}\{G^t[\nabla\cdot(\mathbf{K}^{bb}\cdot\nabla)\\
&+\nabla\cdot(\mathbf{K}^{bt}\cdot\nabla)]+G^b[\nabla\cdot(\mathbf{K}^{tb}\cdot\nabla)+\nabla\cdot(\mathbf{K}^{tt}\cdot\nabla)]\}\\
&+\frac{1}{G^b\gamma^t+G^t\gamma^b}\left\{\gamma^b\frac{\partial}{\partial t}[\nabla\cdot(\mathbf{K}^{tt}\cdot\nabla)]+\gamma^t\frac{\partial}{\partial t}[\nabla\cdot(\mathbf{K}^{bb}\cdot\nabla)]\right\}T^i\\
&+\frac{1}{G^b\gamma^t+G^t\gamma^b}\left\{[\nabla\cdot(\mathbf{K}^{bt}\cdot\nabla)][\nabla\cdot(\mathbf{K}^{tb}\cdot\nabla)]-[\nabla\cdot(\mathbf{K}^{bb}\cdot\nabla)][\nabla\cdot(\mathbf{K}^{tt}\cdot\nabla)]\right.\\
&+\left[\gamma^b\frac{\partial}{\partial t}(\mathbf{u}^{tt}\cdot\nabla)+\gamma^t\frac{\partial}{\partial t}(\mathbf{u}^{bb}\cdot\nabla)\right]-\gamma^b\gamma^t\frac{\partial}{\partial t}(\mathbf{v}^b\cdot\nabla)\\
&+\gamma^b(\mathbf{v}^b\cdot\nabla)[\nabla\cdot(\mathbf{K}^{tt}\cdot\nabla)]+\gamma^b(\mathbf{v}^b\cdot\nabla)(\mathbf{u}^{tt}\cdot\nabla)-[(\mathbf{u}^{bb}\cdot\nabla)(\mathbf{u}^{tt}\cdot\nabla)\\
&-(\mathbf{u}^{bt}\cdot\nabla)(\mathbf{u}^{tb}\cdot\nabla)]-\{(\mathbf{u}^{bb}\cdot\nabla)[\nabla\cdot(\mathbf{K}^{tt}\cdot\nabla)]+(\mathbf{u}^{tt}\cdot\nabla)[\nabla\cdot(\mathbf{K}^{bb}\cdot\nabla)]\\
&\left.-(\mathbf{u}^{bt}\cdot\nabla)[\nabla\cdot(\mathbf{K}^{tb}\cdot\nabla)]-(\mathbf{u}^{tb}\cdot\nabla)[\nabla\cdot(\mathbf{K}^{bt}\cdot\nabla)]\}\right\}T^i+\frac{1}{G^b\gamma^t+G^t\gamma^b}\mathbf{H}^i\varepsilon^t\Phi^t,
\end{aligned}\tag{37}$$

where the index i can take b or t. $\mathbf{H}_i$ takes the form:

$$\mathbf{H}^b=\mathbf{u}^{bt}\cdot\nabla+\nabla\cdot(\mathbf{K}^{bt}\cdot\nabla)+G^b,\tag{38}$$

$$\mathbf{H}^t=\gamma^b\left[\frac{\partial}{\partial t}+(\mathbf{v}^b\cdot\nabla)\right]-\mathbf{u}^{bb}\cdot\nabla-\nabla\cdot(\mathbf{K}^{bb}\cdot\nabla)+G^b.\tag{39}$$

When the system is isotropic and the physical properties of the two phases are constant, it reduces to

$$\begin{aligned}
&\frac{\partial T^i}{\partial t}+\tau_q\frac{\partial^2 T^i}{\partial t^2}+\frac{1}{G^b\gamma^t+G^t\gamma^b}[G^t\gamma^b\mathbf{v}^b-G^b(\mathbf{u}^{tt}+\mathbf{u}^{tb})-G^t(\mathbf{u}^{bb}+\mathbf{u}^{bt})]\cdot\nabla T^i\\
&=\alpha\Delta T^i+\alpha\tau_T\frac{\partial}{\partial t}(\Delta T^i)+\frac{\alpha}{k_e}\left[F(\mathbf{r},t)+\tau_q\frac{\partial F(\mathbf{r},t)}{\partial t}\right]^i,
\end{aligned}\tag{40}$$

where

$$\tau_q=\frac{\gamma^b\gamma^t}{G^b\gamma^t+G^t\gamma^b},\tag{41}$$

$$\tau_T=\frac{\gamma^b k^{tt}+\gamma^t k^{bb}}{k_e},\tag{42}$$

$$\alpha=\frac{k_e}{\rho c}=\frac{k_e}{G^b\gamma^t+G^t\gamma^b},\tag{43}$$

$$k_e=G^t(k^{bb}+k^{bt})+G^b(k^{tb}+k^{tt}),\tag{44}$$

$$\begin{aligned}
\left[F(\mathbf{r},t)+\tau_q\frac{\partial F(\mathbf{r},t)}{\partial t}\right]^i=&\left\{(k^{bt}k^{tb}-k^{bb}k^{tt})\Delta^2+\left[\gamma^b\frac{\partial}{\partial t}(\mathbf{u}^{tt}\cdot\nabla)+\gamma^t\frac{\partial}{\partial t}(\mathbf{u}^{bb}\cdot\nabla)\right]\right.\\
&-\gamma^b\gamma^t\frac{\partial}{\partial t}(\mathbf{v}^b\cdot\nabla)+\gamma^b k^{tt}\Delta(\mathbf{v}^b\cdot\nabla)+\gamma^b(\mathbf{v}^b\cdot\nabla)(\mathbf{u}^{tt}\cdot\nabla)\\
&-[(\mathbf{u}^{bb}\cdot\nabla)(\mathbf{u}^{tt}\cdot\nabla)-(\mathbf{u}^{bt}\cdot\nabla)(\mathbf{u}^{tb}\cdot\nabla)]\\
&-[k^{tt}\Delta(\mathbf{u}^{bb}\cdot\nabla)+k^{bb}\Delta(\mathbf{u}^{tt}\cdot\nabla)-k^{tb}\Delta(\mathbf{u}^{bt}\cdot\nabla)\\
&\left.-k^{bt}\Delta(\mathbf{u}^{tb}\cdot\nabla)]\right\}T^i+h_i\varepsilon^t\Phi^t,
\end{aligned}\tag{45}$$

where

$$h^b=\mathbf{u}^{bt}\cdot\nabla+k^{bt}\Delta+G^b,\tag{46}$$

$$h^t=\gamma^b\left[\frac{\partial}{\partial t}+(\mathbf{v}^b\cdot\nabla)\right]-\mathbf{u}^{bb}\cdot\nabla-k^{bb}\Delta+G^b.\tag{47}$$

This is a dual-phase-lagging heat conduction equation with τ_q and τ_T as the phase lags of the heat flux and the temperature gradient, respectively [8,9,16]. Here, $F(\mathbf{r},t)$ is the volumetric heat source. k_e, ρc, and α are the effective thermal conductivity, volumetric heat capacity and diffusivity, respectively. They depend not only on the volume fractions and the properties of the two phases but also on the microstructure in biological tissues. Although the heat conduction in blood and tissue is assumed to be Fourier-type at the microscale, it is a DPL-type at the macroscale. This feature also occurs in heat-conduction processes of general two-phase systems, identified in [24] that considers pure heat conduction without convection and offers no closures.

The bioheat Eq. (40) differs from those in the literature mainly on: (1) blood or tissue temperature is as the sole unknown temperature, (2) blood–tissue coupled conduction is included, and (3) all the parameters are expressed in terms of properties of blood and tissue and their coupling factor. The readers are referred to [8] for analytical solutions of DPL equation in various systems that can be applied for examining detailed features of bioheat transport in many real problems.

It is interesting to note that the non-traditional convective terms $-\mathbf{u}^{bb}\cdot\nabla T^b-\mathbf{u}^{bt}\cdot\nabla T^t$ and $-\mathbf{u}^{tb}\cdot\nabla T^b-\mathbf{u}^{tt}\cdot\nabla T^t$ in Eqs. (1) and (2) do lead to the non-traditional convective terms $\mathbf{u}^{bb}\cdot\nabla T^i$, $\mathbf{u}^{tb}\cdot\nabla T^i$, $\mathbf{u}^{bt}\cdot\nabla T^i$ and $\mathbf{u}^{tt}\cdot\nabla T^i$ in Eqs. (37) and (40). The velocity-like terms also appear in the source terms of Eqs. (37) and (40).

Furthermore, the heat source $\varepsilon^t\Phi^t$ (which may come from the metabolic reaction in the tissue or external heat supply) and the convective term $\mathbf{v}^b\cdot\nabla T^i$ appear in both the blood and the tissue energy equations [Eqs. (37) and (40)]. Therefore, they are with their macroscale manifestation in both blood and tissue. The blood–tissue interaction generates a very rich way that the blood–tissue interfacial convective heat transfer, the blood velocity, the blood perfusion and the thermal source in tissue affect T^b and T^t [Eqs. (40) and (45)]. It would be very difficult to model these rich interactions by the mixture theory of continuum mechanics.

Consider

$$\frac{\tau_T}{\tau_q}=1+\frac{(\gamma^b)^2G^tk^{tt}+(\gamma^t)^2G^bk^{bb}-\gamma^b\gamma^t(G^tk^{bt}+G^bk^{tb})}{\gamma^b\gamma^tk_e}.\tag{48}$$

It could be larger, equal or smaller than 1 depending on the sign of $(\gamma^b)^2G^tk^{tt}+(\gamma^t)^2G^bk^{bb}-\gamma^b\gamma^t(G^tk^{bt}+G^bk^{tb})$. By the condition for the existence of thermal waves that requires $\tau_T/\tau_q<1$ [8,25], we may have thermal waves in bioheat transport when

$$\begin{aligned}
&(\gamma^b)^2G^tk^{tt}+(\gamma^t)^2G^bk^{bb}-\gamma^b\gamma^t(G^tk^{bt}+G^bk^{tb})\\
&=\left(\gamma^b\sqrt{G^tk^{tt}}-\gamma^t\sqrt{G^bk^{bb}}\right)^2+\gamma^b\gamma^t\left(2\sqrt{G^bk^{bb}G^tk^{tt}}-G^tk^{bt}-G^bk^{tb}\right)<0.
\end{aligned}\tag{49}$$

A necessary (but not sufficient) condition for Eq. (49) is $G^tk^{bt}+G^bk^{tb}>2\sqrt{G^bk^{bb}G^tk^{tt}}$. When the coupling thermal conductivity term k^{bt} and k^{tb} are excluded so that τ_T/τ_q is always larger than 1, thermal waves would not appear. Moreover, there is a time-dependent source term $F(\mathbf{r},t)$ in the DPL macroscale bioheat equation [Eqs. (37) and (40)]. Therefore, the resonance can also occur.

4. Concluding remarks

In an attempt to accurately describe heat transport in biological tissues, we have developed a closed macroscale bioheat model that takes the blood–tissue coupled conduction into account and is with blood or tissue temperature as the sole unknown temperature. The result shows: (i) the dual-phase-lagging bioheat transport at macroscale for both blood and tissue phases, and (ii) the sophisticated effect of the interfacial convective heat transfer, the blood velocity, the perfusion and the metabolic reaction on macroscale temperature fields in blood and tissue.

The dual-phase-lagging heat transport differs from the classical Fourier heat transport mainly on its existence of thermal waves and possible resonance. Such waves and resonance comes from the blood–tissue coupled conduction and will vary features of heat

transport significantly. Focused future efforts are required to find the correlation between the microscale physics of biological tissues and macroscale properties of bioheat transport based on the three closures and to detail properties of thermal waves and how they interact with the heat diffusion and the convection based on the energy equations for blood and tissue macroscale temperatures.

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